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AUTHOR(S): Joseph N. Ginocchio, T-5

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Los Alamos

Los Alamos National Laboratory
Los Alamos, New Mexico 87545

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FERMION DYNAMICAL SYMMETRY AND THE NUCLEAR SHELL MODEL

JOSEPH N. GINOCCHIO

Theoretical Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545 U.S.A.

1. INTRODUCTION

The interacting boson model[1] (IBM) has been very successful in giving a unified and simple description of the spectroscopic properties of a wide range of nuclei, from vibrational through rotational nuclei. The three basic assumptions of the model are that 1) the valence nucleons move about a doubly closed core, 2) the collective low-lying states are composed primarily of coherent pairs of neutrons and pairs of protons coupled to angular momentum zero and two and 3) these coherent pairs are approximated as bosons.

In this review we shall show how it is possible to have fermion Hamiltonians which have a class of collective eigenstates composed entirely of monopole and quadrupole pairs of fermions.[2,3] Hence these models satisfy the assumptions 1) and 2) above but no boson approximation need be made! Thus the Pauli principle is kept in tact.

Furthermore the fermion shell model states excluded in the IBM can be classified by the number of fermion pairs which are not coherent monopole or quadrupole pairs. Hence the mixing of these states into the low-lying spectrum can be calculated in a systematic and tractable manner. Thus we can introduce features which are outside the IBM.

2. MONOPOLE AND QUADRUPOLE PAIRING

Our goal is to construct fermion shell model Hamiltonians which have a class of eigenstates composed of monopole, $J^{\pi}=0^{+}$, pairs and quadrupole, $J^{\pi}=2^{+}$, pairs only. The way to do this is to separate the single-nucleon angular momentum $\vec{j}$ into a pseudo-orbital angular momentum $\vec{k}$ and a pseudo-spin $\vec{i}$.[2,3] We call these "pseudo" because k may not correspond to the real orbital angular momentum of the shell and the spin may be greater than $\frac{1}{2}$. An example is the s-d shell which of course has orbital angular momentum $\ell=0,2$ and spin $s=\frac{1}{2}$. However we can span these states with $k=1$ and $i=3/2$. After the separation, the special subspace is defined by summing over the pseudo-angular momentum or spin thereby making those degrees of freedom inactive. Hence this technique is a way of reducing the number of active degrees of freedom in a fermion shell model and separating the large fermion shell model space into two parts. Another way of looking at this separation is to think of it as a generalization of pairing. In pairing the monopole pair is the only special pair and the single-nucleon angular momenta in this pair are completely coupled to total angular momentum zero. In the present model the single-nucleon angular momentum is split into two parts. Most of the single-nucleon angular momentum is coupled to zero in the special pairs, but a small part of it is not.

To be more explicit we define a nucleon creation operator as $a^{\dagger}_{(ki)jm}$ which creates a nucleon in an orbit with single-nucleon angular momentum j, projection m with pseudo-orbital angular momentum k and pseudo-spin i which are coupled to j,

$$\vec{k} + \vec{i} = \vec{j} \quad . \tag{2.1}$$

A pair of nucleons is then a linear combination of orbitals coupled in k-i coupling to a total pseudo-orbital angular momentum K, pseudo-spin I, with these then coupled to total angular momentum J and projection M,

$$P^{\dagger}_{(KI)JM} = \Sigma_{ki}\ C^{(KI)J}_{ki} [a^{\dagger}_{ki} a^{\dagger}_{ki}]^{(KI)J}_{M} \tag{2.2}$$

Because of antisymmetry, the sum of angular moments is even:

$$K+I+J \text{ even} \quad . \tag{2.3}$$

For our purposes we want to separate out one special angular momentum zero pair and one special angular momentum two pair for which we can construct shell model Hamiltonians which will have a class of eigenstates composed only of these pairs. There are only two ways to do this:

$$\text{A)} \quad C^{(KI)J}_{ki} = \delta_{k,1}\ \delta_{I,0}\ (2i+1)^{\frac{1}{2}} \tag{2.4}$$

$$\text{B)} \quad C^{(KI)J}_{ki} = \delta_{i,\frac{3}{2}}\ \delta_{K,0}\ (2k+1)^{\frac{1}{2}} \quad . \tag{2.5}$$

In the first (second) case as long as the shell model Hamiltonian perserves the pairing of the pseudo-spin (pseudo-orbital angular momentum) the pairs with total pseudo-spin (pseudo-orbital angular momentum) zero will not mix with other pairs. Further, since k=1 ($i=\frac{3}{2}$) and because of the antisymmetrization which leads to equation (2.3), only $J^{\pi}=0^{+},2^{+}$ are allowed for these special pairs.

These two possibilities each lead to Hamiltonians with dynamical symmetries. The first option A leads to an Sp_6 dynamical symmetry; the second option B leads to an SO_8 dynamical symmetry.[3] Each of these models has interesting features. The Sp_6 model has an SU_3 subgroup which means that axially symmetric rotational nuclei emerge from this model when the Hamiltonian has this SU_3 as a dynamical symmetry. On the other hand the SO_8 model has an SO_6 subgroup which gives γ-unstable rotational nuclei when the Hamiltonian has an SO_6 dynamical symmetry. The IBM has both of these possibilities.

However of these two fermion models only the SO_8 model has a one-to-one correspondence between the space spanned by the fermion states composed of the special monopole and quadrupole pairs of neutrons and protons and the space spanned by the monopole and quadrupole bosons.[3] In the Sp_6 model many of the most collective states vanish due to the Pauli principle. For this reason the SO_8 model has received the most attention to date, and we shall discuss that model in sections 3-5 in detail first. However many interesting features appear in the Sp_6 model as well, and there has been a revival of interest of late in this model[4]. We shall report recent developments in this model in section 5.

3. The SO_8 Model

The total number of valence shell model orbits in the SO_8 model can be as large as necessary and is given by

$$2\,\Omega = 4 \sum_k (2k+1) \,, \tag{3.1}$$

and hence the total number of possible states for n valence nucleons can be large, $\binom{2\Omega}{n}$, where n is the number of valence nucleons. A wonderful aspect about the SO_8 model is that all the states in this space can be classified according to irreducible representations of the SO_8 group.[3] In particular the states in the space can be classified according to the number of nucleons in the states, u, not coupled to the special monopole and quadrupole pair. This quantum number is a generalization of the seniority quantum number[8] which just counts the number of nucleons not coupled to a monopole pair. The states with u=0 correspond to the collective subspace composed only of monopole and quadrupole pairs, and has a one-to-one correspondence with the IBM space. The states with u=2 are those with only one pair which is not a monopole or quadrupole pair and so on. This feature means that the study of the coupling of the collective monopole and quadrupole space to the other states left out of the IBM space can be studied in a systematic way.

For odd nuclei u will be odd. The state u=1 correspond to the states of the interacting boson-fermion model[6] in which an odd fermion is coupled to the even-even core described by the IBM. the allowed quantum numbers of u=1 in the SO_8 model have been worked out.[3]

The monopole pair creation operator, $S^\dagger$, and quadrupole pair creation operator, $D^\dagger_\mu$, μ=2,1,0,-1,-2, are given by applying (2.2) and (2.5),

$$S^\dagger = \sum_k (2k+1)^{\frac{1}{2}} [a^\dagger_{k\frac{3}{2}} a^\dagger_{k\frac{3}{2}}]^{(00)0}_0 \tag{3.2a}$$

$$D^\dagger_\mu = \sum_k (2k+1)^{\frac{1}{2}} [a^\dagger_{k\frac{3}{2}} a^\dagger_{k\frac{3}{2}}]^{(02)2}_\mu \quad . \tag{3.2b}$$

These pair creation operators and their hermitian conjugates, plus the multipole operators with total pseudo-orbital angular momentum rank equal to zero,

$$R^{(r)}_\mu = \sum_k (2k+1)^{\frac{1}{2}} [a^\dagger_{k\frac{3}{2}} \tilde{a}_{k\frac{3}{2}}]^{(0,r)r}_\mu ; \; r=0,1,2,3 \tag{3.2c}$$

are the generators of the SO_8 group.[3] In particular, the pseudo-spin generator is

$$\hat{I}_\mu = \sqrt{5}\, R^{(1)}_\mu \quad . \tag{3.2d}$$

In addition to these operators, the multipole operators

$$T^{(t)}_{\mu;k} = [a^\dagger_{k\frac{3}{2}} \tilde{a}_{k\frac{3}{2}}]^{(t,0)t}_\mu ; \; t \text{ odd} \tag{3.3}$$

commute with $S^\dagger$, $D^\dagger$, $R^{(r)}$, and generate an $SO_{(2k+1)}$ group.

Hence any shell model nuclear hamiltonian which has an $SO_8 \otimes \pi_k SO_{(2k+1)}$ dynamical symmetry will have a subspace of eigenstates consisting of $S^\dagger$, $D^\dagger$ pairs only. The most general shell model Hamiltonian of this form will have monopole and quadrupole pairing and multipole interactions:

$$H = G_0 S^\dagger S + G_2 D^\dagger \cdot D + \sum_{r=1,2,3} \kappa^{(r)} R^{(r)} \cdot R^{(r)} \tag{3.4}$$

$$+ \sum_{t\ \mathrm{odd}} \nu_{k'k}^{(t)} T_{k'}^{(t)} \cdot T_{k}^{(t)} + \sum_{r=1,3} \alpha_k^{(r)} (T_k^{(r)} \cdot R^{(r)} + R^{(r)} \cdot T_k^{(r)})$$

where $G_0 < G_2$ and $\kappa^{(r)}$, $\alpha_k^{(r)}$, and $\nu_{k'k}^{(t)}$ are the strengths of the multipole interactions.

The eigenstates of this Hamiltonian will be labeled by the quantum number u. Those with u=0 will correspond to the IBM states and those with u=1 will correspond to the IBFM states. However all shell model states will appear; the remaining states will have a higher value of u.

The group SO_8 has three subgroups chains which have the total pseudo-spin as an SO_3 subgroup. For values of the parameters of the Hamiltonian which conserve the symmetry of these subgroups, the eigenenergies of the Hamiltonian can be given in closed form.

The first symmetry corresponds to subgroup chain

$$SO_8 \supset SO_5 \otimes SU_2 \supset SO_3 \quad . \tag{3.5}$$

In this chain the SO_5 group is the symmetry group of the quadrupole oscillator, SU_2 is the well known quasi-spin group of pairing[7], and SO_3 is the pseudo-spin rotational group. For u=0 states the total pseudo-orbital angular momentum is zero and hence the total angular momentum equals the total pseudo-spin, J=I. This symmetry occurs for $\kappa^{(2)}=G_2$ and the excited energy eigenvalues for u=0 are given by

$$E_5^*(v,\tau,J) = \frac{(G_2-G_0)}{4} v(2\Omega-v+2)+(\kappa^{(3)}-G_2)\tau(\tau+3)+\frac{1}{5}(\kappa^{(1)}-\kappa^{(3)})J(J+1) \quad . \tag{3.6}$$

The quantum number v is the usual seniority,[8]

$$v = n,\ n-2,\ \ldots,\ 0, \tag{3.7}$$

τ is the SO_5 quantum number,

$$\tau = \tfrac{1}{2}v,\ \tfrac{1}{2}v-2,\ \ldots,\ 0 \text{ or } 1 \tag{3.8}$$

and J is the angular momentum with allowed values determined by partitioning τ,

$$\tau = 3p + \lambda \tag{3.9}$$

where p, λ are non-negative integers, and then

$$J = \lambda,\ \lambda+1,\ \ldots,\ 2\lambda-2,\ 2\lambda \quad . \tag{3.10}$$

This spectrum is that of an anharmonic quadrupole oscillator with the energy spacing between levels almost linear in v with anharmonicity from the Pauli principle coming in naturally. This symmetry in the SO_8 model corresponds to the SU_5 symmetry in the IBM.

Another group chain is

$$SO_8 \supset SO_6 \supset SO_5 \supset SO_3 \tag{3.11}$$

and occurs for the pairing strength $G_0=G_2$. The eigenspectrum for u=0 is then

$$E_6^{\star}(\sigma,\tau,J) = (\kappa^{(2)}- G_0)(\sigma-N)(\sigma+N+4) + (\kappa^{(3)}-\kappa^{(2)})\, \tau(\tau+3)$$

$$+ \frac{1}{5}(\kappa^{(1)} -\kappa^{(3)})J(J+1) \tag{3.12}$$

where $N = \frac{1}{2}n$ is the number of pairs of valence nucleons, σ is the SO_6 quantum number,

$$\sigma = N, N-2, \dots 0 \text{ or } 1 \tag{3.13}$$

and the allowed values of τ are

$$\tau = \sigma, \sigma -1, \dots, 0 , \tag{3.14}$$

and the allowed values of the angular momentum J are the same as in (3.9) and (3.10). This symmetry corresponds to a γ-unstable rotor and also corresponds to the SO_6 limit of the IBM.

The final group chain is

$$SO_8 \supset SO_7 \supset SO_5 \supset SO_3 \tag{3.15}$$

and occurs for $\kappa^{(2)} = G_0$. The eigenspectrum for u=0 is given by

$$E_7^{\star}(\bar{v},\tau,J) = (G_2-G_0)\ \frac{\bar{v}}{4}(2\Omega-2n+\bar{v}+10) + (\kappa^{(3)}-G_2)\tau(\tau+3)$$

$$+ \frac{1}{5}(\kappa^{(1)} -\kappa^{(3)})J(J+1)\ . \tag{3.16}$$

This symmetry corresponds to a repulsive quadrupole pairing interaction. The quantum number $\bar{v}$ has the same allowed values as seniority v,

$$\bar{v} = n, n-2, \dots , 0 \tag{3.17}$$

and the allowed values of τ are,

$$\tau = \tfrac{1}{2}\bar{v},\ \tfrac{1}{2}\bar{v}-2,\ \ldots,\ 0 \text{ or } 1 \tag{3.18}$$

and the allowed values of J are the same as in (3.9) and (3.10). The spectrum for a given valence number is that of a anharmonic quadrupole oscillator like the pairing limit, but unlike the pairing limit the spacing between levels decreases as the number of valence nucleons increases.[3)]

For the general Hamiltonian in which none of these three symmetries prevail the spectrum will depend on the relative strength of the pairing interaction and the quadrupole interaction. However it is clear from these solvable limits that a wide variety of spectra can occur in this model.

For the allowed representations of SO_8 and SO_6 for states with u>0, see Reference 3.

An application to the Samarium isotopes in which neutrons and protons were distinguished was successfully carried out in Reference 8.

4. THE Sp_6 MODEL

Just as in the case of the SO_8 model, all the states in this space can be classified according to irreducible representations of the Sp_6 group, and the quantum number u which is the number of nucleons not in the special monopole or quadrupole pair. However unlike the SO_8 model the number of states for u=0 are not in one-to-one correspondence with the IBM[3)]. The number of u=0 states will be less than the number of IBM states because of the Pauli principle. For this reason, which may be unjustified, this model was not studied as much as the SO_8 model. However there has been recent renewed interest in this model[4)].

The monopole pair creation operator, $S^\dagger$, and quadrupole pair creation operator, $\bar{D}^\dagger_\mu$, μ=2,1,0,-1,-2, are given by applying (2.2) and (2.4)

$$S^\dagger = \tfrac{1}{2}\sum_i[3(2i+1)]^{\frac{1}{2}}\,[a^\dagger_{1i}\,a^\dagger_{1i}]^{(00)0}_0 \tag{4.2a}$$

$$\bar{D}^\dagger_\mu = \tfrac{1}{2}\sum_i[3(2i+1)]^{\frac{1}{2}}\,[a^\dagger_{1i}\,a^\dagger_{1i}]^{(20)2}_\mu \ . \tag{4.2b}$$

These pair creation operators and their hermitian conjugates, plus the multipole operators with total pseudo-spin rank equal to zero,

$$\bar{R}^{(r)}_\mu = -\tfrac{1}{2}\sum_i[3(2i+1)]^{\frac{1}{2}}\,[a^\dagger_{1i}\,a_{1i}]^{(r,0)r}_\mu;\ r=0,1,2 \tag{4.2c}$$

are the generators of the Sp_6 group.[3)] In particular the pseudo-orbital angular momentum operator is

$$\hat{K} = -2[2/3]^{\frac{1}{2}} \bar{R}_{\mu}^{(1)} \tag{4.2d}$$

In addition to these operators, the multipole operators

$$\bar{T}^{(t)}_{\mu;i} = \tfrac{1}{2}[3(2i+1)]^{\frac{1}{2}}[a^{\dagger}_{1i}\ \tilde{a}_{1i}\ ]^{(0,t)t}_{\mu};\ t \text{ odd} \tag{4.3}$$

commute with $S^{\dagger}$, $\bar{D}^{\dagger}$, $\bar{R}^{(r)}$ and generate an Sp_{2i+1} group.

Hence any shell model Hamiltonian which has an $Sp_6 \otimes \prod_i Sp_{2i+1}$ dynamical symmetry will have a subspace of eigenstates consisting of $S^{\dagger}$, $\bar{D}^{\dagger}$ pairs only. The most general shell model Hamiltonian of this form will have monopole and quadrupole pairing and multipole interactions:

$$H = G_0 S^{\dagger}S + G_2 \bar{D}^{\dagger}\cdot\bar{D} + \sum_{r=1,2} \kappa^{(r)}\bar{R}^{(r)}\cdot \bar{R}^{(r)} \tag{4.4}$$

$$+ \sum_{t\ \text{odd},\ i',i} \nu^{(t)}_{i'i}\ \bar{T}^{(t)}_{i'}\cdot \bar{T}^{(t)}_{i} + \sum_i \alpha_i(\bar{T}^{(1)}_i \cdot \bar{R}^{(1)} + \bar{R}^{(1)} \cdot \bar{T}^{(1)}_i)$$

where $G_0 < G_2$ and $\kappa^{(r)}$, α_i, and $\nu^{(t)}_{i'i}$ are the strengths of the multipole interactions.

The eigenstates of this Hamiltonian will be labeled by the quantum number u. Those with u=0 will correspond to a subset of the IBM states and those with u=1 will correspond to a subset of the IBFM states. However all shell model states will appear; the remaining states will have a higher value of u.

The group Sp_6 has two subgroups chains which have the total pseudo-orbital angular momentum as an SO_3 subgroup. For values of the parameters of the Hamiltonian which conserve the symmetry of these subgroups, the eigenenergies of the Hamiltonian can be given in closed form.

The first symmetry corresponds to subgroup chain

$$Sp_6 \supset SO_3 \otimes SU_2 \supset SO_3\ . \tag{4.5}$$

In this chain SU_2 is the well known quasi-spin group of pairing[7], and SO_3 is the pseudo-orbital angular momentum group. For u=0 states the pseudo-spin is equal to zero and hence the total angular momentum equals the total pseudo-orbital angular momentum. This symmetry occurs for $\kappa^{(2)}=G_2$ and the excited energy eigenvalues for u=0 are given by

$$E^{\star}_2(\nu,J) = \frac{(G_2-G_0)}{4}\ \nu(2\Omega-\nu+2) + \tfrac{3}{8}(\kappa^{(1)} - G_2)J(J+1)\ . \tag{4.6}$$

The quantum number v is the usual seniority,[6]

$$v = n, n-2, \ldots, 0, \tag{4.7}$$

and J is the angular momentum. This spectrum is that of an anharmonic oscillator with the energy spacing between levels almost linear in v with anharmonicity from the Pauli principle coming in naturally. This symmetry in the Sp_6 model corresponds partially to the SU_5 symmetry in the IBM. Since Sp_6 has no SO_5 subgroup, there is no τ quantum number as in the SO_8 model. The IBM does have an SO_5 subgroup and it is for this reason that there is no one-to-one correspondence between the Sp_6 model and the IBM.

Another group chain is

$$Sp_6 \supset SU_3 \supset SO_3 \tag{4.8}$$

and occurs for the pairing strength $G_0=G_2$. We use the fact that the SU_3 Casimir operator is

$$C_3 = 2 \sum_{r=1,2} \bar{R}^{(r)} \cdot \bar{R}^{(r)} . \tag{4.9}$$

The eigenspectrum for u=0 is then

$$E_3^*(\lambda,\mu,J) = \frac{(\kappa^{(2)}-G_2)[(\lambda-2N)(\lambda+2N+3)+\mu(\lambda+\mu+3)]}{2} + \frac{3}{8}(\kappa^{(1)}-\kappa^{(2)})J(J+1) , \tag{4.10}$$

where $N = \frac{1}{2}n$ is the number of pairs of valence nucleons, and (λ,μ) are the SU_3 quantum numbers. The allowed values of J for a given representation follow the same rules as in the IBM.[1] This symmetry corresponds to an axially symmetric rotor.

5.1 THE SU_3 GROUND STATE BAND

All the states in the SU_3 representation $(\lambda,\mu) = (2N,0)$ which will correspond to the ground state band for an axially symmetric rotor can be projected from an intrinsic state composed of N intrinsic pairs of nucleons. These intrinsic pairs create two nucleons with pseudo-orbital angular momentum projection zero, but total pseudo-spin zero.

$$A^{\dagger} = \tfrac{1}{2}\sum_i \left[\frac{3(2i+1)}{\Omega}\right]^{\frac{1}{2}} \{a^{\dagger}_{10;i} a^{\dagger}_{10;i}\}^{(0)} \quad , \tag{5.1}$$

where the {} coupling is for pseudo-spin only. Hence this pair does not have a definite pseudo-orbital angular momentum.

For $N \lesssim \Omega/2$, i.e. the half-filled shell, the SU_3 eigenstates will be projected from this intrinsic pair condensate,

$$|(2N,0)K,M;l = 0\rangle = \frac{\sqrt{2K+1}}{8\pi^2 \eta_{NK}} \int d\omega\, D^{(K)}_{M0}(\omega) R_K(\omega) (A^{\dagger})^N |0\rangle \tag{5.2a}$$

where $D^{(K)}_{Mm}(\omega)$ is the Wigner D-function[8], ω are the Euler angles, $R_K(\omega)$ is a pseudo-orbital angular momentum rotation, and η_{NK} is the normalization

$$\eta_{NK} = B_{NK} P_N \tag{5.2b}$$

where B_{NK} is the IBM normalization

$$B_{NK} = \left[\frac{(2N)!N!}{(2N+K+1)!!(2N-K)!!}\right]^{\frac{1}{2}} \quad , \tag{5.2c}$$

P_N is the Pauli correction factor

$$P_N = \left[\frac{(\frac{\Omega}{3}-1)!}{(\frac{\Omega}{3}-N)!(\frac{\Omega}{3})^{N-1}}\right]^{\frac{1}{2}} \tag{5.2d}$$

and $|0\rangle$ is the core.

For $N > \frac{\Omega}{2}$ the SU_3 representation $(2\bar{N},0)$ with $\bar{N} = \Omega - N$ will be lowest in energy and is projected from an intrinsic state of $\bar{N}$ instrinsic pairs of nucleon holes. The vacuum $|0\rangle \rightarrow |\bar{0}\rangle$, the closed shell, and $N \rightarrow \bar{N}$ in the formulae (5.2).

From (5.2d) we see that $P_N = 0$ for $N > \frac{\Omega}{3}$. Hence in this case the $(2N,0)$ representation vanishes because of the Pauli principle. Likewise $(2\bar{N},0)$ vanishes for $\bar{N} > \frac{\Omega}{3}$. Thus these lowest SU_3 representations do not exist in the Sp_6 model for

$$\frac{\Omega}{3} < N < \frac{2\Omega}{3} \quad . \tag{5.3}$$

This is an example of states which do not exist in the Sp_6 because of the Pauli principle but do exist in the IBM. This may not be a defect; only by comparison with data can we judge whether this is a valid effect which exists in nuclei.[10]

5.2 THE SU_3 EXCITED BANDS

We can define an excited u=2 band by replacing one of the pairs in (5.2a) by an intrinsic pair with pseudo-spin I≠0

$$A^\dagger_{I\mu} = \frac{1}{2}\sum_i \left[\frac{3(2i+1)}{\Omega}\right]^{\frac{1}{2}} \{a^\dagger_{10;i} a^\dagger_{10;i}\}^{(I)}_\mu \tag{5.4}$$

Because of antisymmetry, I must of course be even. The u=2 states with SU_3 symmetry are projected from an intrinsic state with N-1 I=0 pairs (5.1) and one pair with I≠0:

$$|(2N,0)K,M;I,\mu\rangle = \frac{\sqrt{2K+1}}{8\pi^2 \eta_{NKI}} \int d\omega\, D^{(K)}_{MO}(\omega) R_K(\omega) (A^\dagger)^{N+1} A^\dagger_{I\mu}|0\rangle \tag{5.5}$$

where

$$\eta_{NKI} = \left[\frac{\frac{\Omega}{3}-N}{N\,\frac{\Omega}{3}-1}\right]^{\frac{1}{2}} \eta_{NK} \frac{(1+(-1)^I)}{2} \quad . \tag{5.6}$$

Hence we see that, as long as I is even but larger than zero, the normalization is independent of I. Furthermore we see that this SU_3 band does not exist for u=2 for

$$\frac{\Omega}{3} \le N \le \frac{2\Omega}{3} \tag{5.7}$$

which is more restrictive than for the u=0 states as shown by (5.3).

The SU_3 representation (2N,0) given in (5.5) occurs for many bands,

$$I = 2i_{max} - 1,\ 2i_{max} - 3,\ \dots\ ,\ 2 \tag{5.8}$$

where i_{max} is the maximum pseudo-spin in the system.

These excited bands are important in understanding backbending in nuclear high spin states.[10]

5.3 STRONGLY COUPLED u=2 BAND

In the u=2 bands described by (5.5), the pseudo-spin is not part of the collective rotational moton. We can define a strongly coupled band by rotating the pseudo-orbital angular momentum and pseudo-spin together:

$$|(2N,0)I;JM\rangle = \frac{\sqrt{2J+1}}{8\pi^2\bar{\eta}_{NIJ}}\int d\omega\, D^{(J)}_{M0}(\omega)R(\omega)A^{\dagger}_{I,\mu=0}(A^{\dagger})^{N-1}|0\rangle \quad . \tag{5.9a}$$

where

$$\bar{\eta}_{NIJ} = \left[\sum_{K}(2K+1)\begin{pmatrix} J & I & K \\ 0 & 0 & 0 \end{pmatrix}^2 \eta^2_{NKI}\right]^{\frac{1}{2}} \quad . \tag{5.9b}$$

In the above the rotation $R(\Omega)$ acts on <u>both</u> pseudo-orbital angular momentum and pseudo-spin.

5.4 TRANSITION RATES

For the quadrupole transitions between u=0 states, the quadrupole operator will be proportional to the quadrupole operator which is a scalar with respect to pseudo-spin; i.e. the operator $\bar{R}^{(2)}_{\mu}$ given in (4.2c). The matrix elements of these operators are the same as those given by the SU_3 limit of the IBM[1]. This must be so because in both cases the quadrupole operator is a generator of the SU_3 group and hence the matrix elements will depend only on the SU_3 quantum number.

5.5 PAIRING ENERGY

The pairing binding energy in the u=0 lowest SU_3 band is

$$\langle(2N,0)K,M;I=0|S^{\dagger}S|(2N,0)K,M;I=0\rangle$$
$$= \frac{(\frac{\Omega}{3}-N+1)(2N-K)(2N+K+1)}{2(2N-1)} \quad . \tag{5.10a}$$

As the angular momentum, J=K, increases, this binding energy decreases. For the u≠0 band the pairing is less which makes these bands higher in energy for the same K:

$$\langle(2N,0)K,M;I\neq0,\mu|S^{\dagger}S|(2N,0)K,M;I\neq0,\mu\rangle$$
$$= \frac{(N-1)}{N}\frac{(\frac{\Omega}{3}-N)}{(\frac{\Omega}{3}-N+1)}\langle(2N,0)K,M;I=0|S^{\dagger}S|(2N,0)K,M;I=0\rangle \quad . \tag{5.10b}$$

However for a given total angular momentum J, where of course $J=I+K, I+K-1, \ldots, |I-K|$, if more angular momentum is put into the pseudo-spin, I, and less into the pseudo-orbital angular momentum K, the pairing binding energy for that state will increase. Hence for some J, the state with $I \neq 0$ may become lower in energy. Thus the Yrast level will be in another band leading to a different moment of inertia for the Yrast band.[10] Of course this effect, which occurs naturally in this model, is outside the scope of the pure IBM since it deals only with the u=0 band. Additional quasi-particle states must be introduced into the IBM.[11]

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